Question 1. You are given $\mu(x) = 0.8x^3$. Determine the value of $\Pr[K(1) = 1]$.

Question 2. The future lifetime of (10) is subject to de Moivre's law and $\mathring{e}_{10} = 50$. Calculate the exact probability that the future lifetime is within one standard deviation of its expectation.

Question 3. You are given $q_{60} = 0.2$ and $q_{61} = 0.3$. Determine the probability that (60) will die between ages 60.5 and 61.5 under uniform distribution of deaths over each year of age.

Question 4. You are given 3 mortality assumptions:

(i) Illustrative Life Table (ILT),
(ii) Constant force model (CF), where $s(x) = e^{-\mu x}$, $x \geq 0$,
(iii) De Moivre model (DM), where $s(x) = 1 - x/\omega,$, $0 \leq x \leq \omega$, $\omega \geq 72$.

For the constant force and De Moivre models, ${}_2p_{70}$ is the same as for the Illustrative Life Table. Rank $e_{70:\overline{2}|}$ for these 3 models.

(You need to show $e_{70:\overline{2}|}$ for all three models.)

Question 5. Given that $q_x = 0.200$ and that the uniform assumption applies to the year of age x to $x+1$, calculate m_x.

Question 6. Given:

(i) The force of mortality $\mu_x(t)$ for $0 \leq t \leq 1$ changes to $\mu_x(t) - c$ where c is a constant.

(ii) Before the change $q_x = 0.05$.

(iii) After the change $q_x = 0.07$.

Calculate c.

Question 7. For a given life age 30, it is estimated that an impact of a medical breakthrough will be an increase of 4 years in $\overset{\circ}{e}_{30}$, the complete expectation of life. Prior to the medical breakthrough, s(x) follow de Moivre's law with $\omega = 100$ as the limiting age. Assuming de Moivre's law still applies after the medical breakthrough, calculate the new limiting age.

Question 8. If the constant force of mortality is given with $\mu = 0.05$, determine the average number of years lived between ages x and $x+1$ by those of the survivorship group who die between those ages.

Question 9. You are given $p_8 = 0.5$ and $e_{8:\overline{9}|} = 5$. Determine the value of $e_{9:\overline{8}|}$.

ACT 3130 Actuarial Models 1
Test 1 Solution

1.

$$ {}_tp_x = e^{-\int_x^{x+t}\mu(s)\,ds} = e^{-\int_x^{x+t}0.8s^3\,ds} = e^{-0.2[(x+t)^4-x^4]} $$

$$ \Pr[K(1)=1] = \int_1^2 {}_tp_1\mu(1+t)\,dt = \int_1^2 0.8(1+t)^3 e^{-0.2[(1+t)^4-1]}\,dt = e^{-3} - e^{-16} = 0.0498 $$

2.

$$ f_T(t) = \frac{1}{\omega-10}, \quad 0 \le t < \omega - 10 $$

$$ \mathring{e}_{10} = \frac{\omega-10}{2} = 50 \quad \Rightarrow \quad \omega = 110 $$

$$ \mathrm{Var}(T) = \frac{100^2}{12} = (28.8675)^2 $$

$$ \Pr(50 - 28.8675 \le T \le 50 + 28.8675) = \frac{2(28.8675)}{100} = 0.57735 $$

3.

$$ {}_{0.5|1}q_{60} = \frac{s(60.5)-s(61.5)}{s(60)} = \frac{\frac{s(60)+s(61)}{2} - \frac{s(61)+s(62)}{2}}{s(60)} = \frac{1-{}_2p_{60}}{2} = \frac{1-(0.8)(0.7)}{2} = 0.22 $$

Alternative:

$$ {}_{0.5|1}q_{60} = {}_{0.5}p_{60}\,{}_{0.5}q_{60.5} + p_{60}\,{}_{0.5}q_{61} = (1-0.5q_{60})\frac{(1-0.5)q_{60}}{1-0.5q_{60}} + (1-q_{60})(0.5)q_{61} $$
$$ = (0.5)(0.2) + (0.8)(0.5)(0.3) = 0.22 $$

4. ${}_2p_{70} = \ell_{72}/\ell_{70} = 0.931759$ in the ILT. Solve $e^{-2\mu} = {}_2p_{70}$ for $\mu = 0.0353405$. Solve $(\omega-72)/(\omega-70) = {}_2p_{70}$ for $\omega = 99.308$.

$$ \text{ILT} = e_{70:\overline{2}|} = \frac{\ell_{71}}{\ell_{70}} + \frac{\ell_{72}}{\ell_{70}} = 1.89858 $$

$$ \text{CT} = e^{-\mu} + e^{-2\mu} = 1.89704 $$

$$ \text{DM} = \frac{\omega-71}{\omega-70} + \frac{\omega-72}{\omega-70} = 1.89764 $$

$$ \text{CT} < \text{DM} < \text{ILT} $$

5.

$$ m_x = \frac{\ell_x - \ell_{x+1}}{\int_0^1 \ell_{x+t}\,dt} = \frac{s(x)-s(x+1)}{\int_0^1 t\,s(x) + (1-t)\,s(x+t)\,dt} $$
$$ = \frac{s(x)-s(x+1)}{0.5\,s(x) + s(x+1) - 0.5\,s(x+1)} = \frac{1-p_x}{0.5+0.5\,p_x} = \frac{0.2}{0.9} = 0.2222 $$

Alternative: Use the formula

$$m_x = \frac{q_x}{1 - 0.5\,q_x} = \frac{0.2}{0.9} = 0.2222$$

6. In the "before" scenario,

$$0.95 = p_x = e^{-\int_0^1 \mu_x(t)\,dt}$$

and in the "after" scenario,

$$0.93 = p_x^* = e^{-(\int_0^1 \mu_x(t) - c\,dt)} = p_x e^c.$$

This gives us that $0.93 = 0.95e^c$, and $c = \log(93/95) \approx -0.0213$

7. Since $\overset{\circ}{e}_x = \dfrac{\omega - x}{2}$ assuming the de Moivre's law. We have

$$\overset{\circ}{e}_{30} = \frac{100 - 30}{2} = 35$$
$$\overset{\circ}{e}{}^*_{30} = 35 + 4 = 39$$
$$\overset{\circ}{e}{}^*_{30} = \frac{\omega^* - 30}{2} = 39 \quad \Rightarrow \quad \omega^* = (2)(39) + 30 = 108$$

8. Use the formula in AM Exerxise 3.32a.

$$a(x) = \frac{\left[\dfrac{1 - e^{-\mu}}{\mu}\right] - e^{-\mu}}{1 - e^{-\mu}} = \frac{\left[\dfrac{1 - e^{-0.05}}{0.05}\right] - e^{-0.05}}{1 - e^{-0.05}} = 0.49583.$$

9. Use the recursion formula:

$$e_{8:\overline{9}|} = p_8(1 + e_{9:\overline{8}|})$$
$$5 = (0.5)(1 + e_{9:\overline{8}|}) \quad \Rightarrow \quad e_{9:\overline{8}|} = 9.$$